International Journal of Theoretical Physics, Vol. 24, No. 7, 1985

The Structure of the Nucleon

M. Sekine[1]

Received January 31, 1985

We assume that nucleons are made of quarks which are made of subquarks which are made of more fundamental subquarks, etc. Thus, finally, the proton and the neutron may be composed of an infinite number of pointlike quarks and antiquarks. The limit particle has quantum numbers of spin $J = 1/2$, isospin $I = 1/2$, third component of isospin $I_3 = +1/2$, and fractional electric charge $Q = (1/2)|e|$, where $|e|$ is the electron charge. All quantum numbers are thus just one-half and this fermion will behave as if it was lepton, since the baryon number approaches zero at an infinite sublayer level. Sum rules in lepton-nucleon scattering have been evaluated using this model. The predicted values are not incompatible with the experimental results.

1. INTRODUCTION

To observe the structure of the nucleon, the inclusive leptonic interactions on nucleons have been studied. For example, in deep inelastic electron proton scattering, as the square of the four-momentum transfer, q^2, is increased, more and more structure of the nucleon will be revealed. From the fact that the inelastic scattering cross section falls very much less rapidly than the elastic scattering cross section, the parton model has been proposed (Feynman, 1969; Bjorken and Paschos, 1969). This model implied that the nucleon could be considered as a composite of fractionally charged, pointlike (structureless) constituents, which are called as "parton." With the increase of q^2, the apparent number of partons increases. Thus more and more partons share the momentum of the nucleon and each parton carries a smaller fraction of the momentum. In the limit of $q^2 \to \infty$, the average number of partons becomes infinite. There are some attempts to identify the original valence quarks and gluons with partons (Feynman, 1972). In modern particle physics, this "standard" model has a very strong appeal.

[1]Department of Applied Electronics, Tokyo Institute of Technology, Tokyo, Japan.

0020-7748/85/0700-0701$04.50/0

Nevertheless, from the fact that increasing q^2 resolves each of partons into smaller structures each carrying less longitudinal fraction than the parent, the existence of a further sublayer cannot be excluded (Kogut and Susskind, 1974a, b). The sublayer quark model has been proposed by some authors (Pati and Salam, 1974; Terazawa, Chikashige, and Akama, 1977; Glashow, 1977; Ne'eman, 1979; 't Hooft, 1979; Harari, 1979; Shupe, 1979). The sublayer quarks are called prequarks, preons, subquarks, maons, alphons, quinks, rishons, and quips in these papers. What comes next after those prequarks or subquarks?

Our arguments are based on the assumption that quarks are made up of subquarks which are made up of more fundamental subquarks, etc. Thus u_N and d_N quarks at level N are made up of u_{N+1} and d_{N+1} quarks at level $N+1$, such as $u_N=(u_{N+1}\ u_{N+1}\ d_{N+1})$ and $d_N=(u_{N+1}\ d_{N+1}\ d_{N+1})$ where $N=0, 1, 2, \ldots, \infty$. The proton and the neutron correspond to u_0 and d_0 quarks, respectively. The number of quarks at level N is 3^N. Thus at $N=\infty$, an infinite number of quarks is considered as constituting the nucleon. In the following, this sublayer quark model will be used to evaluate sum rules in lepton-nucleon scattering.

2. SUM RULES IN LEPTON-NUCLEON SCATTERING

The sublayer quark model postulates quantum numbers at level N of $N=0$ to ∞ (Table I). From Table I, it is easily seen that u_∞ and d_∞ are structureless and that the antiquark of u_∞ is the d_∞ quark. Thus, the u_∞ quark is the ultimate particle of the nucleon. Moreover, u_∞ quark will behave as if it was lepton, since the baryon number disappears at $N\to\infty$.

In deep inelastic scattering of charged leptons by nucleons, Bjorken and Paschos (1969) showed the following structure function for spin $-1/2$ partons at very large momentum and energy transfers, q^2 and ν:

$$\lim_{|q^2|, \nu\to\infty} \nu W_2(q^2, \nu)\to F_2(x)=\sum_i Q_i^2 x f_i(x) \qquad (1)$$

where $x=-q^2/2M\nu$, M is the mass of the nucleon, Q_i the charge on the ith parton, and $f_i(x)\, dx$ is the probability of finding that parton with momentum fraction in the range x to $x+dx$. $F_2(x)$ depends solely on x, not q^2. This is Bjorken scaling. At very large q^2, it is well known that Bjorken scaling is violated. At truly asymptotic values of $|a^2|\to\infty$, νW_2 will become a δ function at $x=0$.

Two sum rules, I_1 and I_2 were derived by Callan and Gross (1968) and Gottfried (1968), respectively.

$$I_1=\int_{x_{\min}=0}^{1} F_2(x)\, dx$$

Table I. Quark Quantum Numbers at Level N

Level	Symbol	Combination	Spin and parity J^P	Baryon number B	Isospin I	Isospin quantum number I_3	Charge Q
0	p (proton)	$u_1u_1d_1$	$\frac{1}{2}^+$	1	$\frac{1}{2}$	$\frac{1}{2}$	$+e$
	n (neutron)	$u_1d_1d_1$	$\frac{1}{2}^+$	1	$\frac{1}{2}$	$-\frac{1}{2}$	0
1	u_1	$u_2u_2d_2$	$\frac{1}{2}^+$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}e$
	d_1	$u_sd_2d_2$	$\frac{1}{2}^+$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}e$
$\vdots$							
N	u_N	$u_{N+1}u_{N+1}d_{N+1}$	$\frac{1}{2}^+$	$\frac{1}{3^N}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1+3^N}{2\times3^N}$
	d_N	$u_{N+1}d_{N+1}d_{N+1}$	$\frac{1}{2}^+$	$\frac{1}{3^N}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1-3^N}{2\times3^N}$
$\vdots$							
∞	u_∞	Structureless	$\frac{1}{2}^+$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}e$
	$d_\infty=\bar{u}_\infty$	Structureless	$\frac{1}{2}^+$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}e$

$$= \text{the mean of the charges squared of all partons} \qquad (2)$$

$$I_2 = \int_{x_{\min}=0}^{1} \frac{F_2(x)}{x}\,dx$$

$$= \text{the sum of the charges squared of all partons} \qquad (3)$$

Table II gives a summary of the comparison of the experimental values of the sum rules with the predictions of three models, that is, the simple three-quark model, the Kuti–Weisskopf quark model (1971), and our infinite sublayer quark model. The data were taken from Friedman and Kendall (1972), Anderson (1976, 1977), Perkins (1977), Henry and Lichtenberg (1978), and Williams (1979).

The value of I_1 depends on an integral over the quark momentum distributions and is not very sensitive to the value of $x_{\min}$. The observed

Table II. Comparison of the Sum Rules with the Predictions in Lepton-Nucleon Scattering

	3 quark[a]	3 quark $+SU(3)$ "sea," $\langle N\rangle^{b}\to\infty$	Sublayer quark, $N^{c}\to\infty$	Measurement	$x_{\min}$	q^2 (GeV/c)^2
I_1^{proton}	$\frac{1}{3}$	$\frac{2}{9}+\frac{1}{3\langle N\rangle}, \frac{2}{9}$	$\frac{3^{2N}+3}{4\times 3^{2N}}, \frac{1}{4}$	0.171 ± 0.006	0.004–0.03	1.0–30
I_1^{neutron}	$\frac{2}{9}$	$\frac{2}{9}, \frac{2}{9}$	$\frac{3^{2N}-1}{4\times 3^{2N}}, \frac{1}{4}$	0.137 ± 0.012	0.004–0.03	1.0–30
$I_1^p - I_1^n$	$\frac{1}{9}$	$\frac{1}{3\langle N\rangle}, 0$	$\frac{1}{3^{2N}}, 0$	0.034 ± 0.015	0.004–0.03	1.0–30
I_2^{proton}	1	$\frac{1}{3}+\frac{2\langle N\rangle}{9}, \infty$	$\frac{3^{2N}+3}{4\times 3^{N}}, \infty$	0.739 ± 0.029	0.05	1.0
I_2^{neutron}	$\frac{2}{3}$	$\frac{2\langle N\rangle}{9}, \infty$	$\frac{3^{2N}-1}{4\times 3^{N}}, \infty$	0.592 ± 0.051	0.05	1.0
$I_2^p - I_2^n$	$\frac{1}{3}$	$\frac{1}{3}, \frac{1}{3}$	$\frac{1}{3^N}, 0$	0.147 ± 0.059	0.05	1.0

[a]The 3 quarks and the sea of quark-antiquark pairs are the Kuti-Weisskopf model (1971).
[b]$\langle N\rangle$: Expectation value of number of quarks.
[c]N: Level of sublayer quarks.

value of I_1 is about one-half of the predicted value from the simple three-quark model. On the other hand, assuming the same momentum distribution of all quarks and antiquarks, the Kuti-Weisskopf model is compatible with the measured values. Our sublayer quark model is also better fit to the experimental value of the difference $I_1^{\text{proton}} - I_1^{\text{neutron}}$ than the three-quark model. The quantity I_2 does not depend on the quark momentum distribution but is very sensitive to the cutoff $x_{\min}$. I_2 might diverge as $x_{\min}\to 0$, since the data seem to indicate that $F_2(x)$ is finite as $x\to 0$. The expected value of I_2^{proton} and I_2^{neutron} for both the Kuti-Weisskopf model and our sublayer model could well diverge as $\langle N\rangle\to\infty$ and $N\to\infty$. The difference $I_2^{\text{proton}} - I_2^{\text{neutron}}$ is independent of sea-quark contributions and sensitive only to the valence quarks in the proton and the neutron, and again is half the expected value of 1/3 from both the simple three-quark model and the Kuti-Weisskopf model, while our sublayer quark model shows that the difference of $I_2^{\text{proton}} - I_2^{\text{neutron}}$ is very sensitive to level N and gives zero at $N\to\infty$. It is very interesting to know whether the difference $I_2^{\text{proton}} - I_2^{\text{neutron}}$ would reach 1/3 or 0 in the asymptotic limit, since the experimental value lies just in the middle of 1/3 and 0. This experiment would validate the correctness of the model.

3. CONCLUSION

We assume that the proton and the neutron are finally made of an infinite number of pointlike (structureless) quarks and antiquarks. This particle has all half quantum numbers. Sum rules are evaluated in lepton-nucleon scattering. The result is in good agreement with the experimental value of $I_1^{\text{proton}} - I_1^{\text{neutron}}$. It is desirable to repeat the experiment of $I_2^{\text{proton}} - I_2^{\text{neutron}}$ to test the model.

REFERENCES

Anderson, H. L. et al. (1976). *Physical Review Letters*, **37**, 4.
Anderson, H. L. et al. (1977). *Physical Review Letters*, **38**, 1450.
Bjorken, J. D., and Paschos, E. A. (1969). *Physical Review*, **185**, 1975.
Callan, C. G., and Gross, D. C. (1968). *Physical Review Letters*, **21**, 311.
Feynman, R. P. (1969). *Physical Review Letters*, **23**, 1415.
Feynman, R. P. (1972). *Photon-Hadron Interactions* (Benjamin, New York).
Friedman, J. T., and Kendall, H. W. (1972). *Annual Review of Nuclear Science*, **22**, 203.
Glashow, S. L. (1977). Harvard preprint HUTP-77/A005, unpublished.
Gottfried, K. (1968). *Physical Review Letters*, **18**, 1174.
Harari, H. (1979). *Physics Letters*, **86B**, 83.
Henry, A. W., and Lichtenberg, D. B. (1978). *Reports on Progress in Physics*, **41**, 113.
Kogut, J., and Susskind, L. (1974a). *Physical Review D*, **9**, 697.
Kogut, J., and Susskind, L. (1974b). *Physical Review D*, **9**, 3391.
Kuti, J., and Weisskopf, V. F. (1971). *Physical Review D*, **4**, 3418.
Ne'eman, Y. (1979). *Physics Letters*, **82B**, 69.
Pati, J. C., and Salam, A. (1974). *Physical Review D*, **10**, 275.
Perkins, D. H. (1977). *Reports on Progress in Physics*, **40**, 409.
Shupe, M. A. (1979). *Physics Letters*, **86B**, 87.
Terazawa, H., Chikashige, Y., and Akama, K. (1977). *Physical Review D*, **15**, 480.
't Hooft, G. (1979). Proceedings of the Einstein Centennial Symposium (Jerusalem).
Williams, W. S. (1979). *Reports on Progress in Physics*, **42**, 1661.